

Mathematica 11.3 Integration Test Results

Test results for the 151 problems in "5.2.5 Inverse cosine functions.m"

Problem 5: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{ArcCos}[c x]}{x (d - c^2 d x^2)} dx$$

Optimal (type 4, 71 leaves, 7 steps):

$$\frac{2 (a + b \operatorname{ArcCos}[c x]) \operatorname{ArcTanh}\left[e^{2 i \operatorname{ArcCos}[c x]}\right]}{d} - \frac{i b \operatorname{PolyLog}\left[2, -e^{2 i \operatorname{ArcCos}[c x]}\right]}{2 d} + \frac{i b \operatorname{PolyLog}\left[2, e^{2 i \operatorname{ArcCos}[c x]}\right]}{2 d}$$

Result (type 4, 143 leaves):

$$-\frac{1}{2 d} \left(2 b \operatorname{ArcCos}[c x] \operatorname{Log}\left[1 - e^{i \operatorname{ArcCos}[c x]}\right] + 2 b \operatorname{ArcCos}[c x] \operatorname{Log}\left[1 + e^{i \operatorname{ArcCos}[c x]}\right] - 2 b \operatorname{ArcCos}[c x] \operatorname{Log}\left[1 + e^{2 i \operatorname{ArcCos}[c x]}\right] - 2 a \operatorname{Log}[x] + a \operatorname{Log}\left[1 - c^2 x^2\right] - 2 i b \operatorname{PolyLog}\left[2, -e^{i \operatorname{ArcCos}[c x]}\right] - 2 i b \operatorname{PolyLog}\left[2, e^{i \operatorname{ArcCos}[c x]}\right] + i b \operatorname{PolyLog}\left[2, -e^{2 i \operatorname{ArcCos}[c x]}\right] \right)$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCos}[c x])}{f + g x} dx$$

Optimal (type 4, 1064 leaves, 29 steps):

$$\begin{aligned}
 & - \frac{a d (c f - g) (c f + g) \sqrt{d - c^2 d x^2}}{g^3} + \frac{b c d x \sqrt{d - c^2 d x^2}}{3 g \sqrt{1 - c^2 x^2}} - \frac{b c d (c f - g) (c f + g) x \sqrt{d - c^2 d x^2}}{g^3 \sqrt{1 - c^2 x^2}} + \\
 & \frac{b c^3 d f x^2 \sqrt{d - c^2 d x^2}}{4 g^2 \sqrt{1 - c^2 x^2}} - \frac{b c^3 d x^3 \sqrt{d - c^2 d x^2}}{9 g \sqrt{1 - c^2 x^2}} - \frac{b d (c f - g) (c f + g) \sqrt{d - c^2 d x^2} \text{ArcCos}[c x]}{g^3} + \\
 & \frac{c^2 d f x \sqrt{d - c^2 d x^2} (a + b \text{ArcCos}[c x])}{2 g^2} + \frac{d (1 - c^2 x^2) \sqrt{d - c^2 d x^2} (a + b \text{ArcCos}[c x])}{3 g} - \\
 & \frac{c d f \sqrt{d - c^2 d x^2} (a + b \text{ArcCos}[c x])^2}{4 b g^2 \sqrt{1 - c^2 x^2}} + \frac{c d (c f - g) (c f + g) x \sqrt{d - c^2 d x^2} (a + b \text{ArcCos}[c x])^2}{2 b g^3 \sqrt{1 - c^2 x^2}} + \\
 & \frac{d (c^2 f^2 - g^2)^2 \sqrt{d - c^2 d x^2} (a + b \text{ArcCos}[c x])^2}{2 b c g^4 (f + g x) \sqrt{1 - c^2 x^2}} + \\
 & \frac{d (c f - g) (c f + g) \sqrt{1 - c^2 x^2} \sqrt{d - c^2 d x^2} (a + b \text{ArcCos}[c x])^2}{2 b c g^2 (f + g x)} + \\
 & \frac{a d (c^2 f^2 - g^2)^{3/2} \sqrt{d - c^2 d x^2} \text{ArcTan}\left[\frac{g + c^2 f x}{\sqrt{c^2 f^2 - g^2} \sqrt{1 - c^2 x^2}}\right]}{g^4 \sqrt{1 - c^2 x^2}} + \frac{1}{g^4 \sqrt{1 - c^2 x^2}} \\
 & i b d (c^2 f^2 - g^2)^{3/2} \sqrt{d - c^2 d x^2} \text{ArcCos}[c x] \text{Log}\left[1 + \frac{e^{i \text{ArcCos}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right] - \\
 & \frac{1}{g^4 \sqrt{1 - c^2 x^2}} i b d (c^2 f^2 - g^2)^{3/2} \sqrt{d - c^2 d x^2} \text{ArcCos}[c x] \text{Log}\left[1 + \frac{e^{i \text{ArcCos}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right] + \\
 & \frac{b d (c^2 f^2 - g^2)^{3/2} \sqrt{d - c^2 d x^2} \text{PolyLog}\left[2, -\frac{e^{i \text{ArcCos}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{g^4 \sqrt{1 - c^2 x^2}} - \\
 & \frac{b d (c^2 f^2 - g^2)^{3/2} \sqrt{d - c^2 d x^2} \text{PolyLog}\left[2, -\frac{e^{i \text{ArcCos}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{g^4 \sqrt{1 - c^2 x^2}}
 \end{aligned}$$

Result (type 4, 3034 leaves):

$$\begin{aligned}
 & \sqrt{-d (-1 + c^2 x^2)} \left(\frac{a d (-3 c^2 f^2 + 4 g^2)}{3 g^3} + \frac{a c^2 d f x}{2 g^2} - \frac{a c^2 d x^2}{3 g} \right) + \\
 & \frac{a c d^{3/2} f (2 c^2 f^2 - 3 g^2) \text{ArcTan}\left[\frac{c x \sqrt{-d (-1 + c^2 x^2)}}{\sqrt{d} (-1 + c^2 x^2)}\right]}{2 g^4} + \frac{a d^{3/2} (-c^2 f^2 + g^2)^{3/2} \text{Log}[f + g x]}{g^4} - \\
 & \frac{1}{g^4} a d^{3/2} (-c^2 f^2 + g^2)^{3/2} \text{Log}\left[d g + c^2 d f x + \sqrt{d} \sqrt{-c^2 f^2 + g^2} \sqrt{-d (-1 + c^2 x^2)}\right] - \frac{1}{2 g^2} \\
 & b d \sqrt{d (1 - c^2 x^2)} \left(-\frac{2 c g x}{\sqrt{1 - c^2 x^2}} - 2 g \text{ArcCos}[c x] + \frac{c f \text{ArcCos}[c x]^2}{\sqrt{1 - c^2 x^2}} + \frac{1}{\sqrt{-c^2 f^2 + g^2} \sqrt{1 - c^2 x^2}} \right)
 \end{aligned}$$

$$\begin{aligned}
 & 2(-cf+g)(cf+g) \left(2 \operatorname{ArcCos}[cx] \operatorname{ArcTanh} \left[\frac{(cf+g) \operatorname{Cot} \left[\frac{1}{2} \operatorname{ArcCos}[cx] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] - \right. \\
 & 2 \operatorname{ArcCos} \left[-\frac{cf}{g} \right] \operatorname{ArcTanh} \left[\frac{(-cf+g) \operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[cx] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] + \\
 & \left(\operatorname{ArcCos} \left[-\frac{cf}{g} \right] - 2i \operatorname{ArcTanh} \left[\frac{(cf+g) \operatorname{Cot} \left[\frac{1}{2} \operatorname{ArcCos}[cx] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] \right) + \\
 & \left. 2i \operatorname{ArcTanh} \left[\frac{(-cf+g) \operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[cx] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] \right) \operatorname{Log} \left[\frac{e^{-\frac{1}{2}i \operatorname{ArcCos}[cx]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{cf+cgx}} \right] + \\
 & \left(\operatorname{ArcCos} \left[-\frac{cf}{g} \right] + 2i \left(\operatorname{ArcTanh} \left[\frac{(cf+g) \operatorname{Cot} \left[\frac{1}{2} \operatorname{ArcCos}[cx] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] - \operatorname{ArcTanh} \left[\right. \right. \right. \\
 & \left. \left. \left. \frac{(-cf+g) \operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[cx] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] \right) \right) \operatorname{Log} \left[\frac{e^{\frac{1}{2}i \operatorname{ArcCos}[cx]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{cf+cgx}} \right] - \\
 & \left(\operatorname{ArcCos} \left[-\frac{cf}{g} \right] - 2i \operatorname{ArcTanh} \left[\frac{(-cf+g) \operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[cx] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] \right) \\
 & \operatorname{Log} \left[\left((cf+g) \left(-icf+ig+\sqrt{-c^2 f^2 + g^2} \right) \left(-i+\operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[cx] \right] \right) \right) \right] / \\
 & \left(g \left(cf+g+\sqrt{-c^2 f^2 + g^2} \operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[cx] \right] \right) \right) - \\
 & \left(\operatorname{ArcCos} \left[-\frac{cf}{g} \right] + 2i \operatorname{ArcTanh} \left[\frac{(-cf+g) \operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[cx] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] \right) \\
 & \operatorname{Log} \left[\left((cf+g) \left(icf-ig+\sqrt{-c^2 f^2 + g^2} \right) \left(i+\operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[cx] \right] \right) \right) \right] / \\
 & \left(g \left(cf+g+\sqrt{-c^2 f^2 + g^2} \operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[cx] \right] \right) \right) + \\
 & i \left(\operatorname{PolyLog} \left[2, \left((cf-i\sqrt{-c^2 f^2 + g^2}) \left(cf+g-\sqrt{-c^2 f^2 + g^2} \operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[cx] \right] \right) \right) \right] / \right. \\
 & \left. \left(g \left(cf+g+\sqrt{-c^2 f^2 + g^2} \operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[cx] \right] \right) \right) \right) - \\
 & \operatorname{PolyLog} \left[2, \left((cf+i\sqrt{-c^2 f^2 + g^2}) \left(cf+g-\sqrt{-c^2 f^2 + g^2} \operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[cx] \right] \right) \right) \right] / \\
 & \left. \left(g \left(cf+g+\sqrt{-c^2 f^2 + g^2} \operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[cx] \right] \right) \right) \right) \right] + \frac{1}{72 \sqrt{1-c^2 x^2}} \\
 & bd \sqrt{d(1-c^2 x^2)} \left(\frac{1}{\sqrt{-c^2 f^2 + g^2}} 9 \left(2 \operatorname{ArcCos}[cx] \operatorname{ArcTanh} \left[\frac{(cf+g) \operatorname{Cot} \left[\frac{1}{2} \operatorname{ArcCos}[cx] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] - \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & 2 \operatorname{ArcCos}\left[-\frac{c f}{g}\right] \operatorname{ArcTanh}\left[\frac{(-c f+g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2+g^2}}\right]+ \\
 & \left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right]-2 i \operatorname{ArcTanh}\left[\frac{(c f+g) \operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2+g^2}}\right]+ \right. \\
 & \left. 2 i \operatorname{ArcTanh}\left[\frac{(-c f+g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2+g^2}}\right]\right) \operatorname{Log}\left[\frac{e^{-\frac{1}{2} i \operatorname{ArcCos}[c x]} \sqrt{-c^2 f^2+g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f+c g x}}\right]+ \\
 & \left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right]+2 i\left(\operatorname{ArcTanh}\left[\frac{(c f+g) \operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2+g^2}}\right]-\operatorname{ArcTanh}\left[\frac{(-c f+g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2+g^2}}\right]\right)\right) \operatorname{Log}\left[\frac{e^{\frac{1}{2} i \operatorname{ArcCos}[c x]} \sqrt{-c^2 f^2+g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f+c g x}}\right]- \\
 & \left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right]-2 i \operatorname{ArcTanh}\left[\frac{(-c f+g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2+g^2}}\right]\right) \\
 & \operatorname{Log}\left[\left((c f+g)\left(-i c f+i g+\sqrt{-c^2 f^2+g^2}\right)\left(-i+\operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right) / \right. \\
 & \left.\left(g\left(c f+g+\sqrt{-c^2 f^2+g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right)\right]- \\
 & \left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right]+2 i \operatorname{ArcTanh}\left[\frac{(-c f+g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2+g^2}}\right]\right) \\
 & \operatorname{Log}\left[\left((c f+g)\left(i c f-i g+\sqrt{-c^2 f^2+g^2}\right)\left(i+\operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right) / \right. \\
 & \left.\left(g\left(c f+g+\sqrt{-c^2 f^2+g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right)\right]+ \\
 & i\left(\operatorname{PolyLog}\left[2,\left(\left(c f-i \sqrt{-c^2 f^2+g^2}\right)\left(c f+g-\sqrt{-c^2 f^2+g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right) / \right. \right. \\
 & \left.\left(g\left(c f+g+\sqrt{-c^2 f^2+g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right)\right)\right]- \\
 & \operatorname{PolyLog}\left[2,\left(\left(c f+i \sqrt{-c^2 f^2+g^2}\right)\left(c f+g-\sqrt{-c^2 f^2+g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right) / \right. \right. \\
 & \left.\left(g\left(c f+g+\sqrt{-c^2 f^2+g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right)\right)\right]+ \\
 & \frac{1}{g^4}\left(18 c g\left(-4 c^2 f^2+g^2\right) x+18 g\left(-4 c^2 f^2+g^2\right) \sqrt{1-c^2 x^2} \operatorname{ArcCos}[c x]+ \right. \\
 & 18 c f\left(2 c^2 f^2-g^2\right) \operatorname{ArcCos}[c x]^2+9 c f g^2 \operatorname{Cos}\left[2 \operatorname{ArcCos}[c x]\right]- \\
 & \left. 2 g^3 \operatorname{Cos}\left[3 \operatorname{ArcCos}[c x]\right]-\frac{1}{\sqrt{-c^2 f^2+g^2}}\right)
 \end{aligned}$$

$$\begin{aligned}
& 9 \left(8 c^4 f^4 - 8 c^2 f^2 g^2 + g^4 \right) \left(2 \operatorname{ArcCos}[c x] \operatorname{ArcTanh}\left[\frac{(c f + g) \operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - \right. \\
& 2 \operatorname{ArcCos}\left[-\frac{c f}{g}\right] \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(c f + g) \operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + 2 i \right. \right. \\
& \left. \left. \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \operatorname{Log}\left[\frac{e^{-\frac{1}{2} i \operatorname{ArcCos}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] + \right. \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 i \left(\operatorname{ArcTanh}\left[\frac{(c f + g) \operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - \right. \right. \right. \\
& \left. \left. \left. \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \operatorname{Log}\left[\frac{e^{\frac{1}{2} i \operatorname{ArcCos}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] - \right. \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \\
& \operatorname{Log}\left[\left((c f + g) \left(-i c f + i g + \sqrt{-c^2 f^2 + g^2} \right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right] \right) \right) / \right. \\
& \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right] \right) \right) \right) - \\
& \left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \\
& \operatorname{Log}\left[\left((c f + g) \left(i c f - i g + \sqrt{-c^2 f^2 + g^2} \right) \left(i + \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right] \right) \right) / \right. \\
& \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right] \right) \right) \right) + \\
& i \left(\operatorname{PolyLog}\left[2, \left((c f - i \sqrt{-c^2 f^2 + g^2}) \left(c f + g - \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right] \right) \right) / \right. \right. \\
& \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right] \right) \right) \right) - \operatorname{PolyLog}\left[2, \right. \\
& \left. \left((c f + i \sqrt{-c^2 f^2 + g^2}) \left(c f + g - \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right] \right) \right) / \right. \\
& \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right] \right) \right) \right) \right) + \\
& \left. 18 c f g^2 \operatorname{ArcCos}[c x] \operatorname{Sin}[2 \operatorname{ArcCos}[c x]] - 6 g^3 \operatorname{ArcCos}[c x] \operatorname{Sin}[3 \operatorname{ArcCos}[c x]] \right) \right)
\end{aligned}$$

Problem 28: Result more than twice size of optimal antiderivative.

$$\int \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCos}[c x])}{f + g x} dx$$

Optimal (type 4, 1637 leaves, 37 steps):

$$\begin{aligned} & \frac{a d^2 (c^2 f^2 - g^2)^2 \sqrt{d - c^2 d x^2}}{g^5} - \frac{2 b c d^2 x \sqrt{d - c^2 d x^2}}{15 g \sqrt{1 - c^2 x^2}} - \frac{b c d^2 (c^2 f^2 - 2 g^2) x \sqrt{d - c^2 d x^2}}{3 g^3 \sqrt{1 - c^2 x^2}} + \\ & \frac{b c d^2 (c^2 f^2 - g^2)^2 x \sqrt{d - c^2 d x^2}}{g^5 \sqrt{1 - c^2 x^2}} + \frac{b c^3 d^2 f x^2 \sqrt{d - c^2 d x^2}}{16 g^2 \sqrt{1 - c^2 x^2}} - \frac{b c^3 d^2 f (c^2 f^2 - 2 g^2) x^2 \sqrt{d - c^2 d x^2}}{4 g^4 \sqrt{1 - c^2 x^2}} - \\ & \frac{b c^3 d^2 x^3 \sqrt{d - c^2 d x^2}}{45 g \sqrt{1 - c^2 x^2}} + \frac{b c^3 d^2 (c^2 f^2 - 2 g^2) x^3 \sqrt{d - c^2 d x^2}}{9 g^3 \sqrt{1 - c^2 x^2}} - \frac{b c^5 d^2 f x^4 \sqrt{d - c^2 d x^2}}{16 g^2 \sqrt{1 - c^2 x^2}} + \\ & \frac{b c^5 d^2 x^5 \sqrt{d - c^2 d x^2}}{25 g \sqrt{1 - c^2 x^2}} + \frac{b d^2 (c^2 f^2 - g^2)^2 \sqrt{d - c^2 d x^2} \operatorname{ArcCos}[c x]}{g^5} + \\ & \frac{c^2 d^2 f x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCos}[c x])}{8 g^2} - \frac{c^2 d^2 f (c^2 f^2 - 2 g^2) x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCos}[c x])}{2 g^4} - \\ & \frac{c^4 d^2 f x^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCos}[c x])}{4 g^2} - \frac{d^2 (1 - c^2 x^2) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCos}[c x])}{3 g} - \\ & \frac{d^2 (c^2 f^2 - 2 g^2) (1 - c^2 x^2) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCos}[c x])}{3 g^3} + \\ & \frac{d^2 (1 - c^2 x^2)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCos}[c x])}{5 g} + \frac{c d^2 f \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCos}[c x])^2}{16 b g^2 \sqrt{1 - c^2 x^2}} + \\ & \frac{c d^2 f (c^2 f^2 - 2 g^2) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCos}[c x])^2}{4 b g^4 \sqrt{1 - c^2 x^2}} - \\ & \frac{c d^2 (c^2 f^2 - g^2)^2 x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCos}[c x])^2}{2 b g^5 \sqrt{1 - c^2 x^2}} - \\ & \frac{d^2 (c^2 f^2 - g^2)^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCos}[c x])^2}{2 b c g^6 (f + g x) \sqrt{1 - c^2 x^2}} - \\ & \frac{d^2 (c^2 f^2 - g^2)^2 \sqrt{1 - c^2 x^2} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCos}[c x])^2}{2 b c g^4 (f + g x)} - \\ & \frac{a d^2 (c^2 f^2 - g^2)^{5/2} \sqrt{d - c^2 d x^2} \operatorname{ArcTan}\left[\frac{g + c^2 f x}{\sqrt{c^2 f^2 - g^2} \sqrt{1 - c^2 x^2}}\right]}{g^6 \sqrt{1 - c^2 x^2}} - \\ & \frac{i b d^2 (c^2 f^2 - g^2)^{5/2} \sqrt{d - c^2 d x^2} \operatorname{ArcCos}[c x] \operatorname{Log}\left[1 + \frac{e^{i \operatorname{ArcCos}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{g^6 \sqrt{1 - c^2 x^2}} + \end{aligned}$$

$$\frac{i b d^2 (c^2 f^2 - g^2)^{5/2} \sqrt{d - c^2 d x^2} \operatorname{ArcCos}[c x] \operatorname{Log}\left[1 + \frac{e^{i \operatorname{ArcCos}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{g^6 \sqrt{1 - c^2 x^2}} -$$

$$\frac{b d^2 (c^2 f^2 - g^2)^{5/2} \sqrt{d - c^2 d x^2} \operatorname{PolyLog}\left[2, -\frac{e^{i \operatorname{ArcCos}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{g^6 \sqrt{1 - c^2 x^2}} +$$

$$\frac{b d^2 (c^2 f^2 - g^2)^{5/2} \sqrt{d - c^2 d x^2} \operatorname{PolyLog}\left[2, -\frac{e^{i \operatorname{ArcCos}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{g^6 \sqrt{1 - c^2 x^2}}$$

Result (type 4, 7206 leaves):

$$\sqrt{-d(-1 + c^2 x^2)} \left(\frac{a d^2 (15 c^4 f^4 - 35 c^2 f^2 g^2 + 23 g^4)}{15 g^5} - \frac{a c^2 d^2 f (4 c^2 f^2 - 9 g^2) x}{8 g^4} - \frac{a c^2 d^2 (-5 c^2 f^2 + 11 g^2) x^2}{15 g^3} - \frac{a c^4 d^2 f x^3}{4 g^2} + \frac{a c^4 d^2 x^4}{5 g} \right) -$$

$$\frac{a c d^{5/2} f (8 c^4 f^4 - 20 c^2 f^2 g^2 + 15 g^4) \operatorname{ArcTan}\left[\frac{c x \sqrt{-d(-1 + c^2 x^2)}}{\sqrt{d}(-1 + c^2 x^2)}\right]}{8 g^6} +$$

$$\frac{a d^{5/2} (-c^2 f^2 + g^2)^{5/2} \operatorname{Log}[f + g x]}{g^6} - \frac{1}{g^6}$$

$$a d^{5/2} (-c^2 f^2 + g^2)^{5/2} \operatorname{Log}[d g + c^2 d f x + \sqrt{d} \sqrt{-c^2 f^2 + g^2} \sqrt{-d(-1 + c^2 x^2)}] - \frac{1}{2 g^2}$$

$$b d^2 \sqrt{d(1 - c^2 x^2)} \left(-\frac{2 c g x}{\sqrt{1 - c^2 x^2}} - 2 g \operatorname{ArcCos}[c x] + \frac{c f \operatorname{ArcCos}[c x]^2}{\sqrt{1 - c^2 x^2}} + \frac{1}{\sqrt{-c^2 f^2 + g^2} \sqrt{1 - c^2 x^2}} \right)$$

$$2(-c f + g)(c f + g) \left(2 \operatorname{ArcCos}[c x] \operatorname{ArcTanh}\left[\frac{(c f + g) \operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - \right.$$

$$\left. 2 \operatorname{ArcCos}\left[-\frac{c f}{g}\right] \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) +$$

$$\left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(c f + g) \operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) +$$

$$\left. 2 i \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \operatorname{Log}\left[\frac{e^{-\frac{1}{2} i \operatorname{ArcCos}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] +$$

$$\left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 i \left(\operatorname{ArcTanh}\left[\frac{(c f + g) \operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right)$$

$$\begin{aligned}
 & \left. \left. \left. \frac{(-cf + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[cx]\right]}{\sqrt{-c^2 f^2 + g^2}} \right) \right) \operatorname{Log}\left[\frac{e^{\frac{1}{2} i \operatorname{ArcCos}[cx]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{cf + cgx}}\right] - \\
 & \left(\operatorname{ArcCos}\left[-\frac{cf}{g}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(-cf + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[cx]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \\
 & \operatorname{Log}\left[\left((cf + g) \left(-i cf + i g + \sqrt{-c^2 f^2 + g^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[cx]\right]\right)\right) \right] / \\
 & \left(g \left(cf + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[cx]\right]\right)\right) - \\
 & \left(\operatorname{ArcCos}\left[-\frac{cf}{g}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(-cf + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[cx]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \\
 & \operatorname{Log}\left[\left((cf + g) \left(i cf - i g + \sqrt{-c^2 f^2 + g^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[cx]\right]\right)\right) \right] / \\
 & \left(g \left(cf + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[cx]\right]\right)\right) + \\
 & i \left(\operatorname{PolyLog}\left[2, \left(\left(cf - i \sqrt{-c^2 f^2 + g^2}\right) \left(cf + g - \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[cx]\right]\right)\right)\right] \right) / \\
 & \left(g \left(cf + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[cx]\right]\right)\right) - \\
 & \operatorname{PolyLog}\left[2, \left(\left(cf + i \sqrt{-c^2 f^2 + g^2}\right) \left(cf + g - \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[cx]\right]\right)\right)\right] / \\
 & \left(g \left(cf + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[cx]\right]\right)\right) \right] \right) + \frac{1}{36 \sqrt{1 - c^2 x^2}} \\
 & b d^2 \sqrt{d(1 - c^2 x^2)} \left(\frac{1}{\sqrt{-c^2 f^2 + g^2}} 9 \left(2 \operatorname{ArcCos}[cx] \operatorname{ArcTanh}\left[\frac{(cf + g) \operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcCos}[cx]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - \right. \right. \\
 & \left. \left. 2 \operatorname{ArcCos}\left[-\frac{cf}{g}\right] \operatorname{ArcTanh}\left[\frac{(-cf + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[cx]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) + \right. \\
 & \left(\operatorname{ArcCos}\left[-\frac{cf}{g}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(cf + g) \operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcCos}[cx]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) + \\
 & \left. 2 i \operatorname{ArcTanh}\left[\frac{(-cf + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[cx]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \operatorname{Log}\left[\frac{e^{-\frac{1}{2} i \operatorname{ArcCos}[cx]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{cf + cgx}}\right] + \\
 & \left(\operatorname{ArcCos}\left[-\frac{cf}{g}\right] + 2 i \left(\operatorname{ArcTanh}\left[\frac{(cf + g) \operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcCos}[cx]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-cf + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[cx]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \operatorname{Log}\left[\frac{e^{\frac{1}{2} i \operatorname{ArcCos}[cx]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{cf + cgx}}\right] -
 \end{aligned}$$

$$\begin{aligned}
 & \left(\text{ArcCos}\left[-\frac{c f}{g}\right] - 2 i \text{ArcTanh}\left[\frac{(-c f + g) \text{Tan}\left[\frac{1}{2} \text{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \\
 & \text{Log}\left[\left((c f + g) \left(-i c f + i g + \sqrt{-c^2 f^2 + g^2}\right) \left(-i + \text{Tan}\left[\frac{1}{2} \text{ArcCos}[c x]\right]\right)\right)\right] / \\
 & \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \text{Tan}\left[\frac{1}{2} \text{ArcCos}[c x]\right]\right)\right) - \\
 & \left(\text{ArcCos}\left[-\frac{c f}{g}\right] + 2 i \text{ArcTanh}\left[\frac{(-c f + g) \text{Tan}\left[\frac{1}{2} \text{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \\
 & \text{Log}\left[\left((c f + g) \left(i c f - i g + \sqrt{-c^2 f^2 + g^2}\right) \left(i + \text{Tan}\left[\frac{1}{2} \text{ArcCos}[c x]\right]\right)\right)\right] / \\
 & \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \text{Tan}\left[\frac{1}{2} \text{ArcCos}[c x]\right]\right)\right) + \\
 & i \left(\text{PolyLog}\left[2, \left(\left(c f - i \sqrt{-c^2 f^2 + g^2}\right) \left(c f + g - \sqrt{-c^2 f^2 + g^2} \text{Tan}\left[\frac{1}{2} \text{ArcCos}[c x]\right]\right)\right)\right] / \right. \\
 & \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \text{Tan}\left[\frac{1}{2} \text{ArcCos}[c x]\right]\right)\right)\right) - \\
 & \text{PolyLog}\left[2, \left(\left(c f + i \sqrt{-c^2 f^2 + g^2}\right) \left(c f + g - \sqrt{-c^2 f^2 + g^2} \text{Tan}\left[\frac{1}{2} \text{ArcCos}[c x]\right]\right)\right)\right] / \right. \\
 & \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \text{Tan}\left[\frac{1}{2} \text{ArcCos}[c x]\right]\right)\right)\right) \right] + \\
 & \frac{1}{g^4} \left(18 c g (-4 c^2 f^2 + g^2) x + 18 g (-4 c^2 f^2 + g^2) \sqrt{1 - c^2 x^2} \text{ArcCos}[c x] + \right. \\
 & 18 c f (2 c^2 f^2 - g^2) \text{ArcCos}[c x]^2 + 9 c f g^2 \text{Cos}[2 \text{ArcCos}[c x]] - \\
 & 2 g^3 \text{Cos}[3 \text{ArcCos}[c x]] - \frac{1}{\sqrt{-c^2 f^2 + g^2}} \\
 & 9 (8 c^4 f^4 - 8 c^2 f^2 g^2 + g^4) \left(2 \text{ArcCos}[c x] \text{ArcTanh}\left[\frac{(c f + g) \text{Cot}\left[\frac{1}{2} \text{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - \right. \\
 & 2 \text{ArcCos}\left[-\frac{c f}{g}\right] \text{ArcTanh}\left[\frac{(-c f + g) \text{Tan}\left[\frac{1}{2} \text{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \\
 & \left. \left(\text{ArcCos}\left[-\frac{c f}{g}\right] - 2 i \text{ArcTanh}\left[\frac{(c f + g) \text{Cot}\left[\frac{1}{2} \text{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + 2 i \right. \right. \\
 & \left. \left. \text{ArcTanh}\left[\frac{(-c f + g) \text{Tan}\left[\frac{1}{2} \text{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \text{Log}\left[\frac{e^{-\frac{1}{2} i \text{ArcCos}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] + \right. \\
 & \left. \left(\text{ArcCos}\left[-\frac{c f}{g}\right] + 2 i \left(\text{ArcTanh}\left[\frac{(c f + g) \text{Cot}\left[\frac{1}{2} \text{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 i \left(\operatorname{ArcTanh}\left[\frac{(c f + g) \operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - \right. \right. \\
 & \quad \left. \left. \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right]\right) \right) \operatorname{Log}\left[\frac{e^{\frac{1}{2} i \operatorname{ArcCos}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] - \\
 & \left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \\
 & \operatorname{Log}\left[\left((c f + g) \left(-i c f + i g + \sqrt{-c^2 f^2 + g^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right) / \right. \\
 & \quad \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right)\right] - \\
 & \left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \\
 & \operatorname{Log}\left[\left((c f + g) \left(i c f - i g + \sqrt{-c^2 f^2 + g^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right) / \right. \\
 & \quad \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right)\right] + \\
 & i \left(\operatorname{PolyLog}\left[2, \left(\left(c f - i \sqrt{-c^2 f^2 + g^2}\right) \left(c f + g - \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right) / \right. \right. \\
 & \quad \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right)\right] - \operatorname{PolyLog}\left[2, \right. \\
 & \quad \left. \left(\left(c f + i \sqrt{-c^2 f^2 + g^2}\right) \left(c f + g - \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right) / \right. \\
 & \quad \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right)\right] \right) \right) + \\
 & \frac{1}{16 \sqrt{-c^2 f^2 + g^2} \sqrt{1 - c^2 x^2}} \sqrt{d (1 - c^2 x^2)} \left(2 \operatorname{ArcCos}[c x] \operatorname{ArcTanh}\left[\frac{(c f + g) \operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - \right. \\
 & 2 \operatorname{ArcCos}\left[-\frac{c f}{g}\right] \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \\
 & \left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(c f + g) \operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) + \\
 & \left. 2 i \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \operatorname{Log}\left[\frac{e^{-\frac{1}{2} i \operatorname{ArcCos}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] +
 \end{aligned}$$

$$\begin{aligned}
 & \left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 i \left(\operatorname{ArcTanh}\left[\frac{(c f + g) \operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - \right. \right. \\
 & \quad \left. \left. \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right]\right) \right) \operatorname{Log}\left[\frac{e^{\frac{1}{2} i \operatorname{ArcCos}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] - \\
 & \left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \\
 & \operatorname{Log}\left[\left((c f + g) \left(-i c f + i g + \sqrt{-c^2 f^2 + g^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right) / \right. \\
 & \quad \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right)\right] - \\
 & \left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \\
 & \operatorname{Log}\left[\left((c f + g) \left(i c f - i g + \sqrt{-c^2 f^2 + g^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right) / \right. \\
 & \quad \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right)\right] + \\
 & i \left(\operatorname{PolyLog}\left[2, \left(\left(c f - i \sqrt{-c^2 f^2 + g^2}\right) \left(c f + g - \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right) / \right. \right. \\
 & \quad \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right)\right] - \right. \\
 & \quad \left. \operatorname{PolyLog}\left[2, \left(\left(c f + i \sqrt{-c^2 f^2 + g^2}\right) \left(c f + g - \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right) / \right. \right. \\
 & \quad \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right)\right] \right) + \\
 & \frac{1}{144 g^4 \sqrt{1 - c^2 x^2}} \sqrt{d (1 - c^2 x^2)} \left(18 c g (-4 c^2 f^2 + g^2) x + 18 g (-4 c^2 f^2 + g^2) \right. \\
 & \quad \left. \sqrt{1 - c^2 x^2} \operatorname{ArcCos}[c x] + 18 c f (2 c^2 f^2 - g^2) \operatorname{ArcCos}[c x]^2 + \right. \\
 & \quad \left. 9 c f g^2 \operatorname{Cos}[2 \operatorname{ArcCos}[c x]] - 2 g^3 \operatorname{Cos}[3 \operatorname{ArcCos}[c x]] - \frac{1}{\sqrt{-c^2 f^2 + g^2}} \right. \\
 & \quad \left. 9 (8 c^4 f^4 - 8 c^2 f^2 g^2 + g^4) \left(2 \operatorname{ArcCos}[c x] \operatorname{ArcTanh}\left[\frac{(c f + g) \operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - \right. \right. \\
 & \quad \left. \left. 2 \operatorname{ArcCos}\left[-\frac{c f}{g}\right] \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left(\text{ArcCos}\left[-\frac{c f}{g}\right] - 2 i \text{ArcTanh}\left[\frac{(c f + g) \text{Cot}\left[\frac{1}{2} \text{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + 2 i \right. \\
 & \quad \left. \text{ArcTanh}\left[\frac{(-c f + g) \text{Tan}\left[\frac{1}{2} \text{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \text{Log}\left[\frac{e^{-\frac{1}{2} i \text{ArcCos}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] + \\
 & \left(\text{ArcCos}\left[-\frac{c f}{g}\right] + 2 i \left(\text{ArcTanh}\left[\frac{(c f + g) \text{Cot}\left[\frac{1}{2} \text{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - \right. \right. \\
 & \quad \left. \left. \text{ArcTanh}\left[\frac{(-c f + g) \text{Tan}\left[\frac{1}{2} \text{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \text{Log}\left[\frac{e^{\frac{1}{2} i \text{ArcCos}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] - \\
 & \left(\text{ArcCos}\left[-\frac{c f}{g}\right] - 2 i \text{ArcTanh}\left[\frac{(-c f + g) \text{Tan}\left[\frac{1}{2} \text{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \\
 & \quad \text{Log}\left[\left((c f + g) \left(-i c f + i g + \sqrt{-c^2 f^2 + g^2} \right) \left(-i + \text{Tan}\left[\frac{1}{2} \text{ArcCos}[c x]\right] \right) \right) / \right. \\
 & \quad \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \text{Tan}\left[\frac{1}{2} \text{ArcCos}[c x]\right] \right) \right) \right] - \\
 & \left(\text{ArcCos}\left[-\frac{c f}{g}\right] + 2 i \text{ArcTanh}\left[\frac{(-c f + g) \text{Tan}\left[\frac{1}{2} \text{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \\
 & \quad \text{Log}\left[\left((c f + g) \left(i c f - i g + \sqrt{-c^2 f^2 + g^2} \right) \left(i + \text{Tan}\left[\frac{1}{2} \text{ArcCos}[c x]\right] \right) \right) / \right. \\
 & \quad \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \text{Tan}\left[\frac{1}{2} \text{ArcCos}[c x]\right] \right) \right) \right] + \\
 & i \left(\text{PolyLog}\left[2, \left((c f - i \sqrt{-c^2 f^2 + g^2}) \left(c f + g - \sqrt{-c^2 f^2 + g^2} \text{Tan}\left[\frac{1}{2} \text{ArcCos}[c x]\right] \right) \right) / \right. \right. \\
 & \quad \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \text{Tan}\left[\frac{1}{2} \text{ArcCos}[c x]\right] \right) \right) \right] - \text{PolyLog}\left[2, \right. \\
 & \quad \left. \left((c f + i \sqrt{-c^2 f^2 + g^2}) \left(c f + g - \sqrt{-c^2 f^2 + g^2} \text{Tan}\left[\frac{1}{2} \text{ArcCos}[c x]\right] \right) \right) / \right. \\
 & \quad \left. \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \text{Tan}\left[\frac{1}{2} \text{ArcCos}[c x]\right] \right) \right) \right) \right] \right) + \\
 & \left. 18 c f g^2 \text{ArcCos}[c x] \text{Sin}[2 \text{ArcCos}[c x]] - 6 g^3 \text{ArcCos}[c x] \text{Sin}[3 \text{ArcCos}[c x]] \right) + \\
 & \frac{1}{32 \sqrt{1 - c^2 x^2}} \sqrt{d (1 - c^2 x^2)} \left(-\frac{2 c (16 c^4 f^4 - 12 c^2 f^2 g^2 + g^4) x}{g^5} - \right. \\
 & \quad \left. \frac{32 c^4 f^4 \sqrt{1 - c^2 x^2} \text{ArcCos}[c x]}{g^5} + \right.
 \end{aligned}$$

$$\begin{aligned}
 & \frac{24 c^2 f^2 \sqrt{1 - c^2 x^2} \operatorname{ArcCos}[c x]}{g^3} - \\
 & \frac{2 \sqrt{1 - c^2 x^2} \operatorname{ArcCos}[c x]}{g} + \frac{16 c^5 f^5 \operatorname{ArcCos}[c x]^2}{g^6} - \\
 & \frac{16 c^3 f^3 \operatorname{ArcCos}[c x]^2}{g^4} + \frac{3 c f \operatorname{ArcCos}[c x]^2}{g^2} - \\
 & \frac{2 c f (-2 c^2 f^2 + g^2) \operatorname{Cos}[2 \operatorname{ArcCos}[c x]]}{g^4} - \\
 & \frac{8 c^2 f^2 \operatorname{Cos}[3 \operatorname{ArcCos}[c x]]}{9 g^3} + \frac{2 \operatorname{Cos}[3 \operatorname{ArcCos}[c x]]}{9 g} + \\
 & \frac{c f \operatorname{Cos}[4 \operatorname{ArcCos}[c x]]}{4 g^2} - \frac{2 \operatorname{Cos}[5 \operatorname{ArcCos}[c x]]}{25 g} + \\
 & \frac{1}{g^6 \sqrt{-c^2 f^2 + g^2}} (-2 c^2 f^2 + g^2) (16 c^4 f^4 - 16 c^2 f^2 g^2 + g^4) \\
 & \left(2 \operatorname{ArcCos}[c x] \operatorname{ArcTanh}\left[\frac{(c f + g) \operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - \right. \\
 & \left. 2 \operatorname{ArcCos}\left[-\frac{c f}{g}\right] \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \right. \\
 & \left. \left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(c f + g) \operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + 2 i \right. \right. \\
 & \left. \left. \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \operatorname{Log}\left[\frac{e^{-\frac{1}{2} i \operatorname{ArcCos}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] + \right. \\
 & \left. \left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 i \left(\operatorname{ArcTanh}\left[\frac{(c f + g) \operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - \right. \right. \\
 & \left. \left. \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right) \operatorname{Log}\left[\frac{e^{\frac{1}{2} i \operatorname{ArcCos}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c f + c g x}}\right] - \\
 & \left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \\
 & \operatorname{Log}\left[\left((c f + g) \left(-i c f + i g + \sqrt{-c^2 f^2 + g^2} \right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right] \right) \right) / \right. \\
 & \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right] \right) \right) \right] - \\
 & \left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left(\text{Log} \left[\left((c f + g) \left(i c f - i g + \sqrt{-c^2 f^2 + g^2} \right) \left(i + \text{Tan} \left[\frac{1}{2} \text{ArcCos} [c x] \right] \right) \right) \right] / \right. \\
 & \quad \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \text{Tan} \left[\frac{1}{2} \text{ArcCos} [c x] \right] \right) \right) + \\
 & \quad i \left(\text{PolyLog} [2, \left((c f - i \sqrt{-c^2 f^2 + g^2}) \left(c f + g - \sqrt{-c^2 f^2 + g^2} \text{Tan} \left[\frac{1}{2} \text{ArcCos} [c x] \right] \right) \right) \right] / \right. \\
 & \quad \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \text{Tan} \left[\frac{1}{2} \text{ArcCos} [c x] \right] \right) \right) - \text{PolyLog} [2, \\
 & \quad \left((c f + i \sqrt{-c^2 f^2 + g^2}) \left(c f + g - \sqrt{-c^2 f^2 + g^2} \text{Tan} \left[\frac{1}{2} \text{ArcCos} [c x] \right] \right) \right) \right] / \\
 & \quad \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \text{Tan} \left[\frac{1}{2} \text{ArcCos} [c x] \right] \right) \right) \right) + \\
 & \frac{8 c^3 f^3 \text{ArcCos} [c x] \text{Sin} [2 \text{ArcCos} [c x]]}{g^4} - \frac{4 c f \text{ArcCos} [c x] \text{Sin} [2 \text{ArcCos} [c x]]}{g^2} - \\
 & \frac{8 c^2 f^2 \text{ArcCos} [c x] \text{Sin} [3 \text{ArcCos} [c x]]}{3 g^3} + \\
 & \frac{2 \text{ArcCos} [c x] \text{Sin} [3 \text{ArcCos} [c x]]}{3 g} + \\
 & \frac{c f \text{ArcCos} [c x] \text{Sin} [4 \text{ArcCos} [c x]]}{g^2} - \\
 & \left. \left. \frac{2 \text{ArcCos} [c x] \text{Sin} [5 \text{ArcCos} [c x]]}{5 g} \right) \right)
 \end{aligned}$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b \text{ArcCos} [c x]}{(f + g x) \sqrt{d - c^2 d x^2}} dx$$

Optimal (type 4, 370 leaves, 10 steps):

$$\begin{aligned}
 & \frac{i \sqrt{1 - c^2 x^2} (a + b \text{ArcCos} [c x]) \text{Log} \left[1 + \frac{e^{i \text{ArcCos} [c x]} g}{c f - \sqrt{c^2 f^2 - g^2}} \right]}{\sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2}} - \\
 & \frac{i \sqrt{1 - c^2 x^2} (a + b \text{ArcCos} [c x]) \text{Log} \left[1 + \frac{e^{i \text{ArcCos} [c x]} g}{c f + \sqrt{c^2 f^2 - g^2}} \right]}{\sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2}} + \\
 & \frac{b \sqrt{1 - c^2 x^2} \text{PolyLog} \left[2, -\frac{e^{i \text{ArcCos} [c x]} g}{c f - \sqrt{c^2 f^2 - g^2}} \right]}{\sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2}} - \frac{b \sqrt{1 - c^2 x^2} \text{PolyLog} \left[2, -\frac{e^{i \text{ArcCos} [c x]} g}{c f + \sqrt{c^2 f^2 - g^2}} \right]}{\sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2}}
 \end{aligned}$$

Result (type 4, 930 leaves):

$$\begin{aligned}
 & \frac{1}{\sqrt{-c^2 f^2 + g^2}} \left(\frac{a \operatorname{Log}[f + g x]}{\sqrt{d}} - \frac{a \operatorname{Log}[d (g + c^2 f x) + \sqrt{d} \sqrt{-c^2 f^2 + g^2} \sqrt{d - c^2 d x^2}]}{\sqrt{d}} \right) - \\
 & \frac{1}{\sqrt{d - c^2 d x^2}} b \sqrt{1 - c^2 x^2} \left(2 \operatorname{ArcCos}[c x] \operatorname{ArcTanh} \left[\frac{(c f + g) \operatorname{Cot} \left[\frac{1}{2} \operatorname{ArcCos}[c x] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] - \right. \\
 & \left. 2 \operatorname{ArcCos} \left[-\frac{c f}{g} \right] \operatorname{ArcTanh} \left[\frac{(-c f + g) \operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[c x] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] + \right. \\
 & \left(\operatorname{ArcCos} \left[-\frac{c f}{g} \right] - 2 i \operatorname{ArcTanh} \left[\frac{(c f + g) \operatorname{Cot} \left[\frac{1}{2} \operatorname{ArcCos}[c x] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] + \right. \\
 & \left. \left. 2 i \operatorname{ArcTanh} \left[\frac{(-c f + g) \operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[c x] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] \right) \operatorname{Log} \left[\frac{e^{-\frac{1}{2} i \operatorname{ArcCos}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c (f + g x)}} \right] + \right. \\
 & \left(\operatorname{ArcCos} \left[-\frac{c f}{g} \right] + 2 i \left(\operatorname{ArcTanh} \left[\frac{(c f + g) \operatorname{Cot} \left[\frac{1}{2} \operatorname{ArcCos}[c x] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] - \right. \right. \\
 & \left. \left. \operatorname{ArcTanh} \left[\frac{(-c f + g) \operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[c x] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] \right) \right) \operatorname{Log} \left[\frac{e^{\frac{1}{2} i \operatorname{ArcCos}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c (f + g x)}} \right] - \\
 & \left(\operatorname{ArcCos} \left[-\frac{c f}{g} \right] - 2 i \operatorname{ArcTanh} \left[\frac{(-c f + g) \operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[c x] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] \right) \\
 & \operatorname{Log} \left[\left((c f + g) \left(-i c f + i g + \sqrt{-c^2 f^2 + g^2} \right) \left(-i + \operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[c x] \right] \right) \right) / \right. \\
 & \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[c x] \right] \right) \right) \right] - \\
 & \left(\operatorname{ArcCos} \left[-\frac{c f}{g} \right] + 2 i \operatorname{ArcTanh} \left[\frac{(-c f + g) \operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[c x] \right]}{\sqrt{-c^2 f^2 + g^2}} \right] \right) \\
 & \operatorname{Log} \left[\left((c f + g) \left(i c f - i g + \sqrt{-c^2 f^2 + g^2} \right) \left(i + \operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[c x] \right] \right) \right) / \right. \\
 & \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[c x] \right] \right) \right) \right] + \\
 & i \left(\operatorname{PolyLog} \left[2, \left((c f - i \sqrt{-c^2 f^2 + g^2}) \left(c f + g - \sqrt{-c^2 f^2 + g^2} \operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[c x] \right] \right) \right) / \right. \right. \\
 & \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[c x] \right] \right) \right) \right] \right) - \\
 & \operatorname{PolyLog} \left[2, \left((c f + i \sqrt{-c^2 f^2 + g^2}) \left(c f + g - \sqrt{-c^2 f^2 + g^2} \operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[c x] \right] \right) \right) / \right. \\
 & \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan} \left[\frac{1}{2} \operatorname{ArcCos}[c x] \right] \right) \right) \right] \right) \right)
 \end{aligned}$$

Problem 33: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{ArcCos}[c x]}{(f + g x)^2 \sqrt{d - c^2 d x^2}} dx$$

Optimal (type 4, 496 leaves, 13 steps):

$$\frac{g (1 - c^2 x^2) (a + b \operatorname{ArcCos}[c x])}{(c^2 f^2 - g^2) (f + g x) \sqrt{d - c^2 d x^2}} + \frac{i c^2 f \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcCos}[c x]) \operatorname{Log}\left[1 + \frac{e^{i \operatorname{ArcCos}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{(c^2 f^2 - g^2)^{3/2} \sqrt{d - c^2 d x^2}} -$$

$$\frac{i c^2 f \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcCos}[c x]) \operatorname{Log}\left[1 + \frac{e^{i \operatorname{ArcCos}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{(c^2 f^2 - g^2)^{3/2} \sqrt{d - c^2 d x^2}} + \frac{b c \sqrt{1 - c^2 x^2} \operatorname{Log}[f + g x]}{(c^2 f^2 - g^2) \sqrt{d - c^2 d x^2}} +$$

$$\frac{b c^2 f \sqrt{1 - c^2 x^2} \operatorname{PolyLog}\left[2, -\frac{e^{i \operatorname{ArcCos}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{(c^2 f^2 - g^2)^{3/2} \sqrt{d - c^2 d x^2}} - \frac{b c^2 f \sqrt{1 - c^2 x^2} \operatorname{PolyLog}\left[2, -\frac{e^{i \operatorname{ArcCos}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{(c^2 f^2 - g^2)^{3/2} \sqrt{d - c^2 d x^2}}$$

Result (type 4, 1108 leaves):

$$-\frac{a g \sqrt{d - c^2 d x^2}}{d (-c^2 f^2 + g^2) (f + g x)} - \frac{a c^2 f \operatorname{Log}[f + g x]}{\sqrt{d} (-c^2 f^2 + g^2)^{3/2}} -$$

$$\frac{a c^2 f \operatorname{Log}\left[d (g + c^2 f x) + \sqrt{d} \sqrt{-c^2 f^2 + g^2} \sqrt{d - c^2 d x^2}\right]}{\sqrt{d} (c f - g) (c f + g) \sqrt{-c^2 f^2 + g^2}} -$$

$$\frac{1}{\sqrt{d - c^2 d x^2}} b c \sqrt{1 - c^2 x^2} \left(-\frac{g \sqrt{1 - c^2 x^2} \operatorname{ArcCos}[c x]}{(c f - g) (c f + g) (c f + c g x)} - \frac{\operatorname{Log}\left[1 + \frac{g x}{f}\right]}{c^2 f^2 - g^2} - \right.$$

$$\frac{1}{(-c^2 f^2 + g^2)^{3/2}} c f \left(2 \operatorname{ArcCos}[c x] \operatorname{ArcTanh}\left[\frac{(c f + g) \operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - \right.$$

$$2 \operatorname{ArcCos}\left[-\frac{c f}{g}\right] \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] +$$

$$\left. \left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(c f + g) \operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] + \right.$$

$$\left. \left. 2 i \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \operatorname{Log}\left[\frac{e^{-\frac{1}{2} i \operatorname{ArcCos}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c (f + g x)}}\right] +$$

$$\left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 i \left(\operatorname{ArcTanh}\left[\frac{(c f + g) \operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \right)$$

$$\begin{aligned}
 & \left. \left. \left. \left. \left. \frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}} \right) \right) \right) \right) \operatorname{Log}\left[\frac{e^{\frac{1}{2} i \operatorname{ArcCos}[c x]} \sqrt{-c^2 f^2 + g^2}}{\sqrt{2} \sqrt{g} \sqrt{c(f + g x)}}\right] - \\
 & \left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \\
 & \operatorname{Log}\left[\left((c f + g) \left(-i c f + i g + \sqrt{-c^2 f^2 + g^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right) / \right. \\
 & \quad \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right)\right] - \\
 & \left(\operatorname{ArcCos}\left[-\frac{c f}{g}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(-c f + g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{-c^2 f^2 + g^2}}\right] \right) \\
 & \operatorname{Log}\left[\left((c f + g) \left(i c f - i g + \sqrt{-c^2 f^2 + g^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right) / \right. \\
 & \quad \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right)\right] + \\
 & i \left(\operatorname{PolyLog}\left[2, \left(\left(c f - i \sqrt{-c^2 f^2 + g^2}\right) \left(c f + g - \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right) / \right. \right. \\
 & \quad \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right)\right] - \right. \\
 & \quad \left. \operatorname{PolyLog}\left[2, \left(\left(c f + i \sqrt{-c^2 f^2 + g^2}\right) \left(c f + g - \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right) / \right. \right. \\
 & \quad \left. \left(g \left(c f + g + \sqrt{-c^2 f^2 + g^2} \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]\right)\right)\right] \right) \right) \right) \right)
 \end{aligned}$$

Problem 35: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcCos}[c x])^2 \operatorname{Log}[h(f + g x)^m]}{\sqrt{1 - c^2 x^2}} dx$$

Optimal (type 4, 496 leaves, 13 steps):

$$\begin{aligned}
 & - \frac{i m (a + b \operatorname{ArcCos}[c x])^4}{12 b^2 c} + \frac{m (a + b \operatorname{ArcCos}[c x])^3 \operatorname{Log}\left[1 + \frac{e^{i \operatorname{ArcCos}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{3 b c} + \\
 & \frac{m (a + b \operatorname{ArcCos}[c x])^3 \operatorname{Log}\left[1 + \frac{e^{i \operatorname{ArcCos}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{3 b c} - \frac{(a + b \operatorname{ArcCos}[c x])^3 \operatorname{Log}[h (f + g x)^m]}{3 b c} - \\
 & \frac{i m (a + b \operatorname{ArcCos}[c x])^2 \operatorname{PolyLog}\left[2, -\frac{e^{i \operatorname{ArcCos}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{c} - \\
 & \frac{i m (a + b \operatorname{ArcCos}[c x])^2 \operatorname{PolyLog}\left[2, -\frac{e^{i \operatorname{ArcCos}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{c} + \\
 & \frac{2 b m (a + b \operatorname{ArcCos}[c x]) \operatorname{PolyLog}\left[3, -\frac{e^{i \operatorname{ArcCos}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{c} + \\
 & \frac{2 b m (a + b \operatorname{ArcCos}[c x]) \operatorname{PolyLog}\left[3, -\frac{e^{i \operatorname{ArcCos}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{c} + \\
 & \frac{2 i b^2 m \operatorname{PolyLog}\left[4, -\frac{e^{i \operatorname{ArcCos}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{c} + \frac{2 i b^2 m \operatorname{PolyLog}\left[4, -\frac{e^{i \operatorname{ArcCos}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{c}
 \end{aligned}$$

Result (type 8, 37 leaves):

$$\int \frac{(a + b \operatorname{ArcCos}[c x])^2 \operatorname{Log}[h (f + g x)^m]}{\sqrt{1 - c^2 x^2}} dx$$

Problem 36: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcCos}[c x]) \operatorname{Log}[h (f + g x)^m]}{\sqrt{1 - c^2 x^2}} dx$$

Optimal (type 4, 374 leaves, 11 steps):

$$\begin{aligned}
 & - \frac{i m (a + b \operatorname{ArcCos}[c x])^3}{6 b^2 c} + \frac{m (a + b \operatorname{ArcCos}[c x])^2 \operatorname{Log}\left[1 + \frac{e^{i \operatorname{ArcCos}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{2 b c} + \\
 & \frac{m (a + b \operatorname{ArcCos}[c x])^2 \operatorname{Log}\left[1 + \frac{e^{i \operatorname{ArcCos}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{2 b c} - \frac{(a + b \operatorname{ArcCos}[c x])^2 \operatorname{Log}[h (f + g x)^m]}{2 b c} - \\
 & \frac{i m (a + b \operatorname{ArcCos}[c x]) \operatorname{PolyLog}\left[2, -\frac{e^{i \operatorname{ArcCos}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{c} - \\
 & \frac{i m (a + b \operatorname{ArcCos}[c x]) \operatorname{PolyLog}\left[2, -\frac{e^{i \operatorname{ArcCos}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{c} + \\
 & \frac{b m \operatorname{PolyLog}\left[3, -\frac{e^{i \operatorname{ArcCos}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{c} + \frac{b m \operatorname{PolyLog}\left[3, -\frac{e^{i \operatorname{ArcCos}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{c}
 \end{aligned}$$

Result (type 4, 1248 leaves):

$$\begin{aligned}
 & \frac{1}{6 c} \left(-3 i a m \operatorname{ArcCos}[c x]^2 - i b m \operatorname{ArcCos}[c x]^3 + \right. \\
 & 24 i a m \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{c f}{g}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(c f - g) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcCos}[c x]\right]}{\sqrt{c^2 f^2 - g^2}}\right] + 3 b m \operatorname{ArcCos}[c x]^2 \\
 & \operatorname{Log}\left[1 + \frac{e^{i \operatorname{ArcCos}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right] + 6 a m \operatorname{ArcCos}[c x] \operatorname{Log}\left[1 + \frac{e^{i \operatorname{ArcCos}[c x]} (c f - \sqrt{c^2 f^2 - g^2})}{g}\right] + \\
 & 3 b m \operatorname{ArcCos}[c x]^2 \operatorname{Log}\left[1 + \frac{e^{i \operatorname{ArcCos}[c x]} (c f - \sqrt{c^2 f^2 - g^2})}{g}\right] + \\
 & 12 a m \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{c f}{g}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{e^{i \operatorname{ArcCos}[c x]} (c f - \sqrt{c^2 f^2 - g^2})}{g}\right] + \\
 & 12 b m \operatorname{ArcCos}[c x] \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{c f}{g}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{e^{i \operatorname{ArcCos}[c x]} (c f - \sqrt{c^2 f^2 - g^2})}{g}\right] + \\
 & \left. 3 b m \operatorname{ArcCos}[c x]^2 \operatorname{Log}\left[1 + \frac{e^{i \operatorname{ArcCos}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right] + \right)
 \end{aligned}$$

$$6 \text{ a m ArcCos}[c x] \operatorname{Log}\left[1 + \frac{e^{i \operatorname{ArcCos}[c x]} \left(c f + \sqrt{c^2 f^2 - g^2}\right)}{g}\right] +$$

$$3 \text{ b m ArcCos}[c x]^2 \operatorname{Log}\left[1 + \frac{e^{i \operatorname{ArcCos}[c x]} \left(c f + \sqrt{c^2 f^2 - g^2}\right)}{g}\right] -$$

$$12 \text{ a m ArcSin}\left[\frac{\sqrt{1 + \frac{c f}{g}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{e^{i \operatorname{ArcCos}[c x]} \left(c f + \sqrt{c^2 f^2 - g^2}\right)}{g}\right] -$$

$$12 \text{ b m ArcCos}[c x] \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{c f}{g}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{e^{i \operatorname{ArcCos}[c x]} \left(c f + \sqrt{c^2 f^2 - g^2}\right)}{g}\right] -$$

$$6 \text{ a m ArcCos}[c x] \operatorname{Log}[f + g x] - 6 \text{ a m ArcSin}[c x] \operatorname{Log}[f + g x] -$$

$$3 \text{ b ArcCos}[c x]^2 \operatorname{Log}[h (f + g x)^m] + 6 \text{ a ArcSin}[c x] \operatorname{Log}[h (f + g x)^m] -$$

$$3 \text{ b m ArcCos}[c x]^2 \operatorname{Log}\left[1 + \frac{\left(c f - \sqrt{c^2 f^2 - g^2}\right) \left(c x + i \sqrt{1 - c^2 x^2}\right)}{g}\right] -$$

$$12 \text{ b m ArcCos}[c x] \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{c f}{g}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{\left(c f - \sqrt{c^2 f^2 - g^2}\right) \left(c x + i \sqrt{1 - c^2 x^2}\right)}{g}\right] -$$

$$3 \text{ b m ArcCos}[c x]^2 \operatorname{Log}\left[1 + \frac{\left(c f + \sqrt{c^2 f^2 - g^2}\right) \left(c x + i \sqrt{1 - c^2 x^2}\right)}{g}\right] +$$

$$12 \text{ b m ArcCos}[c x] \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{c f}{g}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{\left(c f + \sqrt{c^2 f^2 - g^2}\right) \left(c x + i \sqrt{1 - c^2 x^2}\right)}{g}\right] -$$

$$6 \text{ i b m ArcCos}[c x] \operatorname{PolyLog}\left[2, \frac{e^{i \operatorname{ArcCos}[c x]} g}{-c f + \sqrt{c^2 f^2 - g^2}}\right] -$$

$$6 \text{ i a m PolyLog}\left[2, \frac{e^{i \operatorname{ArcCos}[c x]} \left(-c f + \sqrt{c^2 f^2 - g^2}\right)}{g}\right] -$$

$$6 \text{ i b m ArcCos}[c x] \operatorname{PolyLog}\left[2, -\frac{e^{i \operatorname{ArcCos}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right] -$$

$$6 \text{ i a m PolyLog}\left[2, -\frac{e^{i \operatorname{ArcCos}[c x]} \left(c f + \sqrt{c^2 f^2 - g^2}\right)}{g}\right] +$$

$$\left. 6 b m \operatorname{PolyLog}\left[3, \frac{e^{i \operatorname{ArcCos}[c x]} g}{-c f + \sqrt{c^2 f^2 - g^2}}\right] + 6 b m \operatorname{PolyLog}\left[3, -\frac{e^{i \operatorname{ArcCos}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right] \right)$$

Problem 37: Attempted integration timed out after 120 seconds.

$$\int \frac{\operatorname{Log}[h (f + g x)^m]}{\sqrt{1 - c^2 x^2}} dx$$

Optimal (type 4, 237 leaves, 9 steps):

$$\frac{i m \operatorname{ArcSin}[c x]^2}{2 c} - \frac{m \operatorname{ArcSin}[c x] \operatorname{Log}\left[1 - \frac{i e^{i \operatorname{ArcSin}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{c} - \frac{m \operatorname{ArcSin}[c x] \operatorname{Log}\left[1 - \frac{i e^{i \operatorname{ArcSin}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{c} +$$

$$\frac{\operatorname{ArcSin}[c x] \operatorname{Log}[h (f + g x)^m]}{c} + \frac{i m \operatorname{PolyLog}\left[2, \frac{i e^{i \operatorname{ArcSin}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{c} + \frac{i m \operatorname{PolyLog}\left[2, \frac{i e^{i \operatorname{ArcSin}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{c}$$

Result (type 1, 1 leaves):

???

Problem 54: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcCos}[a x]}{(c + d x^2)^{3/2}} dx$$

Optimal (type 3, 66 leaves, 6 steps):

$$\frac{x \operatorname{ArcCos}[a x]}{c \sqrt{c + d x^2}} - \frac{\operatorname{ArcTan}\left[\frac{\sqrt{d} \sqrt{1 - a^2 x^2}}{a \sqrt{c + d x^2}}\right]}{c \sqrt{d}}$$

Result (type 6, 159 leaves):

$$\frac{1}{\sqrt{c + d x^2}} x \left(\left(2 a x \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, a^2 x^2, -\frac{d x^2}{c}\right] \right) / \right.$$

$$\left. \left(\sqrt{1 - a^2 x^2} \left(4 c \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, a^2 x^2, -\frac{d x^2}{c}\right] + x^2 \left(-d \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, a^2 x^2, -\frac{d x^2}{c}\right] + a^2 c \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, a^2 x^2, -\frac{d x^2}{c}\right] \right) \right) \right) \right) + \frac{\operatorname{ArcCos}[a x]}{c}$$

Problem 55: Result unnecessarily involves higher level functions.

$$\int \frac{\text{ArcCos}[a x]}{(c + d x^2)^{5/2}} dx$$

Optimal (type 3, 136 leaves, 7 steps):

$$-\frac{a \sqrt{1 - a^2 x^2}}{3 c (a^2 c + d) \sqrt{c + d x^2}} + \frac{x \text{ArcCos}[a x]}{3 c (c + d x^2)^{3/2}} + \frac{2 x \text{ArcCos}[a x]}{3 c^2 \sqrt{c + d x^2}} - \frac{2 \text{ArcTan}\left[\frac{\sqrt{d} \sqrt{1 - a^2 x^2}}{a \sqrt{c + d x^2}}\right]}{3 c^2 \sqrt{d}}$$

Result (type 6, 216 leaves):

$$\frac{1}{3 c^2 (c + d x^2)^{3/2}} \left(-\frac{a c \sqrt{1 - a^2 x^2} (c + d x^2)}{a^2 c + d} + \left(4 a c x^2 (c + d x^2) \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, a^2 x^2, -\frac{d x^2}{c}\right] \right) / \right. \\ \left. \left(\sqrt{1 - a^2 x^2} \left(4 c \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, a^2 x^2, -\frac{d x^2}{c}\right] + x^2 \left(-d \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, a^2 x^2, -\frac{d x^2}{c}\right] + a^2 c \text{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, a^2 x^2, -\frac{d x^2}{c}\right] \right) \right) \right) \right) + (3 c x + 2 d x^3) \text{ArcCos}[a x]$$

Problem 56: Result unnecessarily involves higher level functions.

$$\int \frac{\text{ArcCos}[a x]}{(c + d x^2)^{7/2}} dx$$

Optimal (type 3, 211 leaves, 8 steps):

$$-\frac{a \sqrt{1 - a^2 x^2}}{15 c (a^2 c + d) (c + d x^2)^{3/2}} - \frac{2 a (3 a^2 c + 2 d) \sqrt{1 - a^2 x^2}}{15 c^2 (a^2 c + d)^2 \sqrt{c + d x^2}} + \\ \frac{x \text{ArcCos}[a x]}{5 c (c + d x^2)^{5/2}} + \frac{4 x \text{ArcCos}[a x]}{15 c^2 (c + d x^2)^{3/2}} + \frac{8 x \text{ArcCos}[a x]}{15 c^3 \sqrt{c + d x^2}} - \frac{8 \text{ArcTan}\left[\frac{\sqrt{d} \sqrt{1 - a^2 x^2}}{a \sqrt{c + d x^2}}\right]}{15 c^3 \sqrt{d}}$$

Result (type 6, 277 leaves):

$$\frac{1}{15 c^3 (c+d x^2)^{5/2}} \left(-\frac{a c^2 \sqrt{1-a^2 x^2} (c+d x^2)}{a^2 c+d} - \frac{2 a c (3 a^2 c+2 d) \sqrt{1-a^2 x^2} (c+d x^2)^2}{(a^2 c+d)^2} + \right. \\ \left. \left(16 a c x^2 (c+d x^2)^2 \text{AppellF1} \left[1, \frac{1}{2}, \frac{1}{2}, 2, a^2 x^2, -\frac{d x^2}{c} \right] \right) / \right. \\ \left. \left(\sqrt{1-a^2 x^2} \left(4 c \text{AppellF1} \left[1, \frac{1}{2}, \frac{1}{2}, 2, a^2 x^2, -\frac{d x^2}{c} \right] + \right. \right. \right. \\ \left. \left. \left. x^2 \left(-d \text{AppellF1} \left[2, \frac{1}{2}, \frac{3}{2}, 3, a^2 x^2, -\frac{d x^2}{c} \right] + a^2 c \text{AppellF1} \left[2, \frac{3}{2}, \frac{1}{2}, 3, a^2 x^2, -\frac{d x^2}{c} \right] \right) \right) \right) \right) + \\ \left. x (15 c^2 + 20 c d x^2 + 8 d^2 x^4) \text{ArcCos}[a x] \right)$$

Problem 81: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \text{ArcCos}[a x^2] dx$$

Optimal (type 4, 55 leaves, 4 steps):

$$-\frac{2 x \sqrt{1-a^2 x^4}}{9 a} + \frac{1}{3} x^3 \text{ArcCos}[a x^2] + \frac{2 \text{EllipticF}[\text{ArcSin}[\sqrt{a} x], -1]}{9 a^{3/2}}$$

Result (type 4, 63 leaves):

$$\frac{1}{9} \left(-\frac{2 x \sqrt{1-a^2 x^4}}{a} + 3 x^3 \text{ArcCos}[a x^2] + \frac{2 i \text{EllipticF}[i \text{ArcSinh}[\sqrt{-a} x], -1]}{(-a)^{3/2}} \right)$$

Problem 83: Result unnecessarily involves imaginary or complex numbers.

$$\int \text{ArcCos}[a x^2] dx$$

Optimal (type 4, 43 leaves, 6 steps):

$$x \text{ArcCos}[a x^2] + \frac{2 \text{EllipticF}[\text{ArcSin}[\sqrt{a} x], -1]}{\sqrt{a}} - \frac{2 \text{EllipticF}[\text{ArcSin}[\sqrt{a} x], -1]}{\sqrt{a}}$$

Result (type 4, 56 leaves):

$$x \text{ArcCos}[a x^2] + \frac{1}{(-a)^{3/2}} \\ 2 i a \left(\text{EllipticE}[i \text{ArcSinh}[\sqrt{-a} x], -1] - \text{EllipticF}[i \text{ArcSinh}[\sqrt{-a} x], -1] \right)$$

Problem 85: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcCos}[a x^2]}{x^2} dx$$

Optimal (type 4, 29 leaves, 3 steps):

$$-\frac{\text{ArcCos}[a x^2]}{x} - 2\sqrt{a} \text{EllipticF}[\text{ArcSin}[\sqrt{a} x], -1]$$

Result (type 4, 40 leaves):

$$-\frac{\text{ArcCos}[a x^2] + 2i\sqrt{-a} x \text{EllipticF}[i \text{ArcSinh}[\sqrt{-a} x], -1]}{x}$$

Problem 88: Result more than twice size of optimal antiderivative.

$$\int \text{ArcCos}\left[\frac{a}{x}\right] dx$$

Optimal (type 3, 27 leaves, 5 steps):

$$x \text{ArcSec}\left[\frac{x}{a}\right] - a \text{ArcTanh}\left[\sqrt{1 - \frac{a^2}{x^2}}\right]$$

Result (type 3, 84 leaves):

$$x \text{ArcCos}\left[\frac{a}{x}\right] - \frac{a\sqrt{-a^2+x^2} \left(-\text{Log}\left[1 - \frac{x}{\sqrt{-a^2+x^2}}\right] + \text{Log}\left[1 + \frac{x}{\sqrt{-a^2+x^2}}\right]\right)}{2\sqrt{1 - \frac{a^2}{x^2}} x}$$

Problem 135: Unable to integrate problem.

$$\int \frac{\left(a + b \text{ArcCos}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^3}{1 - c^2 x^2} dx$$

Optimal (type 4, 279 leaves, 8 steps):

$$\begin{aligned} & \frac{i \left(a + b \text{ArcCos}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^4}{4bc} - \frac{\left(a + b \text{ArcCos}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^3 \text{Log}\left[1 + e^{2i \text{ArcCos}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{c} + \\ & \frac{3ib \left(a + b \text{ArcCos}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2 \text{PolyLog}\left[2, -e^{2i \text{ArcCos}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{2c} - \\ & \frac{3b^2 \left(a + b \text{ArcCos}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right) \text{PolyLog}\left[3, -e^{2i \text{ArcCos}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{2c} - \frac{3ib^3 \text{PolyLog}\left[4, -e^{2i \text{ArcCos}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{4c} \end{aligned}$$

Result (type 8, 42 leaves):

$$\int \frac{\left(a + b \text{ArcCos}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^3}{1 - c^2 x^2} dx$$

Problem 136: Unable to integrate problem.

$$\int \frac{\left(a + b \operatorname{ArcCos}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2}{1-c^2x^2} dx$$

Optimal (type 4, 207 leaves, 7 steps):

$$\frac{i \left(a + b \operatorname{ArcCos}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^3}{3bc} - \frac{\left(a + b \operatorname{ArcCos}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2 \operatorname{Log}\left[1 + e^{2i \operatorname{ArcCos}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{c} +$$

$$\frac{i b \left(a + b \operatorname{ArcCos}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right) \operatorname{PolyLog}\left[2, -e^{2i \operatorname{ArcCos}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{c} - \frac{b^2 \operatorname{PolyLog}\left[3, -e^{2i \operatorname{ArcCos}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{2c}$$

Result (type 8, 42 leaves):

$$\int \frac{\left(a + b \operatorname{ArcCos}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2}{1-c^2x^2} dx$$

Problem 137: Unable to integrate problem.

$$\int \frac{a + b \operatorname{ArcCos}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}{1-c^2x^2} dx$$

Optimal (type 4, 141 leaves, 6 steps):

$$\frac{i \left(a + b \operatorname{ArcCos}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2}{2bc} -$$

$$\frac{\left(a + b \operatorname{ArcCos}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right) \operatorname{Log}\left[1 + e^{2i \operatorname{ArcCos}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{c} + \frac{i b \operatorname{PolyLog}\left[2, -e^{2i \operatorname{ArcCos}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}\right]}{2c}$$

Result (type 8, 40 leaves):

$$\int \frac{a + b \operatorname{ArcCos}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]}{1-c^2x^2} dx$$

Problem 140: Attempted integration timed out after 120 seconds.

$$\int \operatorname{ArcCos}\left[c e^{a+bx}\right] dx$$

Optimal (type 4, 84 leaves, 6 steps):

$$-\frac{i \operatorname{ArcCos}\left[\frac{c e^{a+bx}}{2b}\right]^2}{2b} + \frac{\operatorname{ArcCos}\left[\frac{c e^{a+bx}}{b}\right] \operatorname{Log}\left[1 + e^{2i \operatorname{ArcCos}\left[\frac{c e^{a+bx}}{b}\right]}\right]}{b} - \frac{i \operatorname{PolyLog}\left[2, -e^{2i \operatorname{ArcCos}\left[\frac{c e^{a+bx}}{b}\right]}\right]}{2b}$$

Result (type 1, 1 leaves):

???

Problem 147: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \operatorname{ArcCos}\left[\frac{c}{a+bx}\right] dx$$

Optimal (type 3, 48 leaves, 6 steps):

$$\frac{(a+bx) \operatorname{ArcSec}\left[\frac{a}{c} + \frac{bx}{c}\right]}{b} - \frac{c \operatorname{ArcTanh}\left[\sqrt{1 - \frac{c^2}{(a+bx)^2}}\right]}{b}$$

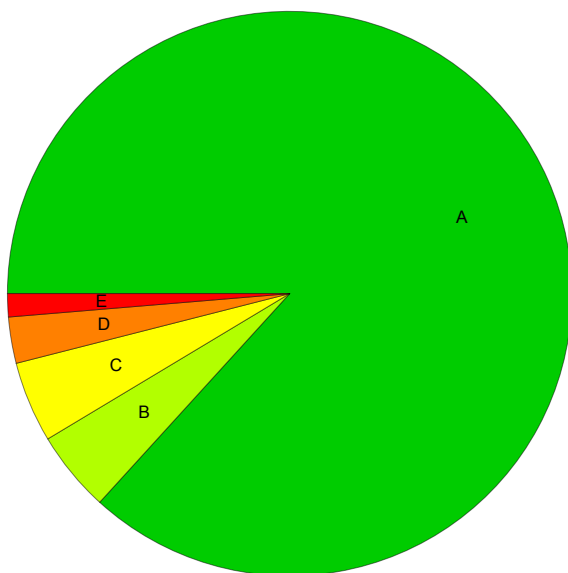
Result (type 3, 167 leaves):

$$x \operatorname{ArcCos}\left[\frac{c}{a+bx}\right] -$$

$$\left((a+bx) \sqrt{\frac{a^2 - c^2 + 2abx + b^2x^2}{(a+bx)^2}} \left(i a \operatorname{Log}\left[-\frac{2b^2(-ic + \sqrt{a^2 - c^2 + 2abx + b^2x^2})}{a(a+bx)}\right] \right) + \right. \\ \left. c \operatorname{Log}\left[a+bx + \sqrt{a^2 - c^2 + 2abx + b^2x^2}\right] \right) / \left(b \sqrt{a^2 - c^2 + 2abx + b^2x^2} \right)$$

Summary of Integration Test Results

151 integration problems



- A - 131 optimal antiderivatives
- B - 7 more than twice size of optimal antiderivatives
- C - 7 unnecessarily complex antiderivatives
- D - 4 unable to integrate problems
- E - 2 integration timeouts